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Forecasting Stock Market Volatility Using GARCH Model : A Case Study of NSE

Gurcharan Singh* and Salony Kansal*

** School of Management Studies, Punjabi University, Patiala*

Abstract

Volatility represents risk and is a great concern for anyone who is dealing with money or investing in the stock market or any other financial instruments. Hence, the issues of volatility have become increasingly important in recent times to financial practitioners, market participants, retail investors, regulators and researchers. The aim of this study is to fit an appropriate GARCH model to estimate the conditional market volatility based on S&P CNX Nifty. This study uses 14 years of daily data (April 1996-March 2010). The findings suggest that stock market has significant ARCH effects and it is appropriate to use GARCH model to estimate the process.

INTRODUCTION

There has been considerable volatility (and uncertainty) in the past few years in mature and emerging financial markets worldwide. Most investors and financial analysts are concerned about the uncertainty of the returns on their investment assets, caused by the variability in speculative market prices (and market risk) and the instability of business performance (Alexander, 1999). Recent developments in financial econometrics require the use of quantitative models that are able to explain the attitude of investors not only towards expected returns and risks, but towards volatility as well. Hence, market participants should be aware of the need to manage risks associated with volatility. This requires models that are capable of dealing with the volatility of the market (and the series). Due to unexpected events, uncertainties in prices (and returns) and the non-constant variance in the financial markets, financial analysts started to model and explain the behaviour of stock market returns and volatility using time series econometric models (Christos, 2008).

STATISTICAL MODEL OF STOCK MARKET VOLATILITY

Broadly, there are four possible approaches for estimating and forecasting volatility. These are : Historical Volatility Models, Implied Volatility Models, Conditional Volatility Models and Extreme Value Estimators.

Historical Volatility Models

This is the simplest model for capturing the volatility and is widely used. It simply involves calculating the variance or standard deviation of returns over some past period. It is a total risk measure that captures both idiosyncratic as well as systematic risk. Standard deviation measures how widely values (closing prices for instance) are dispersed from the average. Dispersion is the difference between the actual value (closing price) and the average value (mean closing price). The larger the difference between the closing prices and the average price, the higher will be the standard deviation and higher the volatility. The closer are the closing prices to the average price, the lower will be the standard deviation and lower the volatility. It is a relative measure, i.e., standard deviation of stock returns in one period can be compared with standard deviation of another period to understand which period has been more volatile. This measure is then used as the volatility forecast for the future period (Srivastava , 2006).

Implied Volatility Models

In Black and Scholes (1973), there are five variables to determine the value of options that include spot price, strike price, time to maturity, interest rate and volatility. Among these variables, all are observed except for volatility that needs to be estimated. The model assumes the constant volatility over the life of option and generally involves the use of historical volatility as an estimate of expected future volatility. Given the market price of options and four other variables, could use backward induction techniques to derive standard deviation that the market used as an input. Such a volatility estimate is called implied volatility.

Conditional Volatility Models

Conditional Volatility Models (ARCH/GARCH) , unlike the traditional or extreme value estimators, take into account the time-varying nature of volatility. The ARCH and the GARCH models assume conditional heteroskedastic with homoskedastic unconditional error variance. That is, the changes in variance are a function of the realizations of preceding errors and these changes represent temporary and random departures from a constant unconditional variance. The advantage of GARCH model is that it captures the tendency in financial data for volatility clustering. It, therefore, enables us to make the connection between information and volatility explicit since any change in the rate of informational arrival to the market will change the volatility in the market (Pandey, 2005).

Extreme Value Estimator

These estimators are similar to traditional estimators except that these also incorporate high and low prices observed unlike traditional estimators which are based on closing prices of the asset (Pandey, 2005).

REVIEW OF LITERATURE

Various studies have taken place on the concept of volatility of stock markets throughout the globe. Some of the important contributions are as follows :

Philip and Dick (1996) studied the performance of the GARCH model and two of its non-linear modifications to forecast weekly stock market volatility. They found that the QGARCH model is best when the estimation sample does not contain extreme observations such as the 1987 stock market crash and that the GJR model cannot be recommended for forecasting.

Padhi (2005) explained the stock market volatility at the individual script level and at the aggregate indices level. The empirical analysis has been done by using ARCH, GARCH model. ARCH is mean model and it is based on daily data for the time period from January 1990 to November 2004. The analysis reveals the same trend of volatility in the case of aggregate indices and five different sectors such as electrical, machinery, mining, non-metallic and power plant sector. The GARCH (1,1) model is persistent for all the five aggregate indices and individual company.

Karamakar (2006) measured the volatility of daily returns in the Indian stock market over the period 1961 to 2005 by using the GARCH model and observed that the market is tranquil and volatile. The level of volatility is modest for the first two decades of the 1960s and 1970s. Almost from the beginning of 1980s there were indications of change in the mood of the market. Volatility touches new high from 1985 and during the year 1992, it surpassed all previous records and continued to increase till the end of the decade. During the last two years, volatility has declined and this period is accompanied by increasing price rise, mainly fuelled by the investment made by the FIIs.

Saryal (2007) used GARCH models to estimate conditional stock market volatility using monthly data from Turkey and Canada. He further analysed the impact of inflation and found that the rate of inflation has high predictive power for stock market volatility in Turkey, whereas it is weaker, but still significant for Canada. The findings suggest that the higher the rate of inflation, the greater the stock market volatility, that is, higher rates of inflation are coincident with greater stock market risk. These results are also supported when the change in the rate of inflation is used as explanatory variable for predicting conditional volatility.

Joshi and Pandya (2008) examined the nature of volatility in the Indian stock market, namely, the BSE. Various volatility estimators and diagnostic tests suggest certain stylized facts about volatility such as volatility clustering and mean reverting. This study uses 16 years of daily data (July 1990-October 2006) on Sensex to capture these facts. ARCH and GARCH models to study the behaviour of volatility and concludes that GARCH (1,1) model satisfactorily explains volatility clustering and its high persistence.

Srivastava Aman (2008) developed models to elucidate the volatility of the stock of the major indices of India i.e. Bombay Stock Exchange (SENSEX) and National Stock Exchange (NIFTY) from April 2000 to March 2008. He used the Autoregressive Conditional Heteroskedasticity (ARCH) models and its extension, the Generalized ARCH, EGARCH and TARCH models to find out the presence of the stock market volatility on Indian stock market. The findings suggest that both the Indian stock exchanges have significant ARCH effects and it is appropriate to use ARCH/GARCH models to estimate the process.

Hung-Chun Liu and Yen-Hsien Lee (2009) investigated how specification of return distribution influences the performance of volatility forecasting using two GARCH models (GARCH-N and GARCH-SGED). Daily spot prices on the Shanghai and Shenzhen composite stock indices provide the empirical sample for discussing and comparing the relative out-of-sample volatility predictive ability, given the growth potential of stock markets in China in the eyes of global investors. Empirical results indicate that the GARCH-SGED model is superior to the GARCH-N model in forecasting China stock markets volatility, for all forecast horizons when model selection is based on MSE or MAE.

Rastogi and Srivastava (2011) used time-varying variance based GARCH process to capture the change in the volatility and studied its impact on the Indian securities market. Further, change in the volatility of US securities market has been compared with change in volatility of Indian securities market at different time of uproar in the Indian economy. It has been found that co-movement in terms of conditional volatility in both the market is not prominent.

OBJECTIVES OF THE STUDY

The objective of the study is to fit an appropriate GARCH model to estimate the conditional market volatility based on S&P CNX Nifty.

DATA AND METHODOLOGY

Scope of the Study

The scope of the study is limited to the past 14 financial years, i.e. 1996 to 2010 data of NSE (S&P CNX Nifty) .The data has been taken as daily closing values of exchange.

Sample Size

We have used S&P CNX Nifty index as a proxy for the stock market because it is the important representative of the Indian Stock Market and also it accounts for two thirds of the turnover.

Data Collection

The study is based mainly on secondary data which have been collected from the official website of NSE.

Statistical Tools

The main tools used in the study are Mean , Standard Deviation, Skewness, Kurtosis, Jarque-Bera, Augmented Dicker Fuller test (ADF) and Granger Causality test.

Measurement of Volatility

Volatility has been measured as standard deviation of the rates of return. The rate of returns have been computed by taking a logarithmic difference of prices of two successive periods. Symbolically, it may be stated as follows :

$r_t = \log_e (p_t/p_{t-1}) = \log_e (p_t) - \log_e(p_{t-1})$ where $\log_e$ is natural logarithm, p_t and p_{t-1} are the closing prices for the two successive periods. The logarithmic difference is symmetric between up and down movements and is expressed in per centage terms.

ARCH and GARCH

Engle (1982) suggests that the conditional variance h_t can be modelled as a function of the lagged ε's, i.e. the predictable volatility is dependent on past news. The most detailed model he develops is the qth order ARCH model, the ARCH (q) :

$$h_t = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \ldots\ldots\ldots + \alpha_q \varepsilon_{t-q}^2$$

where

$\omega, \alpha_1 \ldots\ldots \alpha_q$ = parameters to be estimated

h_t = conditional variance at period t

q = number of lags included in the model

ε_t = innovation in return at time t

where, $\omega > 0, \alpha_1, \alpha_2, \ldots\ldots\ldots, \alpha_q > 0$. In an ARCH (*q*) model, an old news which arrived at the market more than q periods ago has no effect at all on current volatility. Alternatively, if a major market movement occurred yesterday, the day before or up to *q* days ago, the effect will be to increase today's conditional variance.

Bollerslev (1986) generalized the ARCH (*q*) model to the GARCH (*p, q*) model, such that :

$$h_t = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \ldots.. + \alpha_q \varepsilon_{t-q}^2 + \beta_1 h_{t-2} + \ldots.. + \beta_p \varepsilon_{t-p}$$

$\omega, \alpha_1 \ldots\ldots\ldots \alpha_q, \beta_1 \ldots\ldots\ldots\ldots \beta_p$ = parameters to be estimated

where :

h_t = conditional variance at period t

q = number of return innovation lags included in the model

p = number of past volatility lags included in model

ε_t = innovation in return at time t

where $\omega > 0, \alpha_1, \alpha_2, \ldots\ldots\ldots, \alpha_q \geq 0, \beta_1, \beta_2, \ldots\ldots, \beta_p \geq 0$.

The GARCH (p, q) process defined above is stationary when $(\alpha_1 + \alpha_2 + \ldots\ldots + , \alpha_q) + (\beta_1 + \beta_2 + \ldots\ldots + \beta_p) < 1$.

The simplest but often very useful GARCH process is the GARCH (1, 1) process which is also called the genericor 'vanilla' GARCH model given by :

$$h_t = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$$

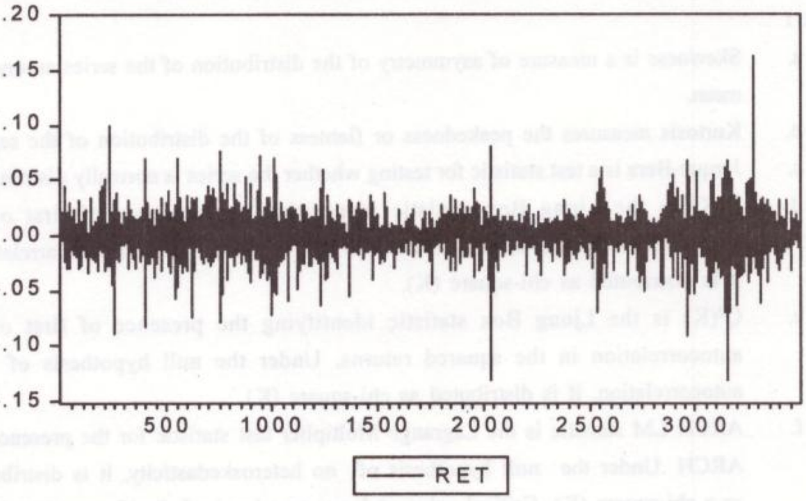
where $\omega > 0, \alpha_1 \geq 0, \beta_1 \geq 0$. The stationary condition for GARCH (1, 1) is $\alpha_1 + \beta_1 < 1$. (Karmakar, 2005)

In the GARCH (1,1) model, the effect of a return shock on current volatility declines geometrically over time. As referred earlier, the GARCH (1, 1) model is found to be an excellent model for a wide range of financial data. The sizes of the parameters α_1 and β_1 determine the short-run dynamics of the resulting volatility time series. Large GARCH lag coefficients β_1 indicate that shocks to conditional variance take a long time to die out, so volatility is 'persistent.' Large GARCH error coefficient α_1 means that volatility reacts quite intensely to market movements and so if α_1 is relatively high and β_1 is relatively low, then volatilities tend to be more 'spiky.' In financial markets, it is common to estimate lag (or 'persistence') coefficients based on daily observation in excess of 0.8 and error (or 'reaction') coefficients not more than 0.2. If $\alpha_1 + \beta_1$ is close to unity, then a 'shock' at time *t* will persist for many future periods. A high value of $\alpha_1 + \beta_1$, therefore, implies a 'long memory.' For $\alpha_1 + \beta_1 = 1$, any shock will lead to a permanent change in all future values of h_t hence shock to the conditional variance is 'persistent.' For $\alpha_1 + \beta_1 = 1$ integrated GARCH process (i.e., IGARCH) is used. For IGARCH, the conditional variance is non-stationary and the unconditional variance is unbounded.

ANALYSIS AND DISCUSSION

A descriptive investigation of the plot of daily returns on S & P CNX Nifty (Figure 1) reveals that returns continuously fluctuated around the mean value that was close to zero. The return measures were both in positive and negative area. More fluctuations be tending to cluster together and were separated by periods of relative calm. This was in agreement with Fama's (1965) observation of "volatility clustering"[1]. From the time series graph of the returns for S&P CNX Nifty, it is analyzed that high volatilities are followed by high volatilities and low volatilities are followed by low volatilities. That means time series have important time varying variances. Additionally, it is appropriate to put conditional variance into the function to clarify the impact of risk on the returns. Hence, GARCH model is the excellent tool for the study.

Figure 1 : Volatility Clustering of Daily Return of S&P CNX Nifty



Source : Based on the data taken from NSE website for selected period.

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1 Volatility Clustering means that large changes in time series tend to be followed by large changes and small changes by small changes.

Table 1
Descriptive Statistics of Daily Returns

	S&P CNX Nifty
Observation Period	April 1996-March 2010
Number of Observation	3493
Mean	0.000476
Median	0.001153
Maximum	0.163343
Minimum	–0.130539
Std. Dev.	0.017524
Skewness	–0.185352
Kurtosis	9.147381
Jarque-Bera	5518.482
Q (1)	12.272 (2-tailed p = 0.00)
Q2 (1)	137.98 (2-tailed p = 0.00)
ARCH LM statistics(at lag = 1)	145.38

Source : Calculated from the data taken from NSE website for selected period.

Notes :

a. Skewness is a measure of asymmetry of the distribution of the series around its mean.

b. Kurtosis measures the peakedness or flatness of the distribution of the series.

c. Jarque-Bera is a test statistic for testing whether the series is normally distributed.

d. Q(K) is the Ljung Box statistic identifying the presence of first order autocorrelation in the returns. Under the null hypothesis of no autocorrelation, it is distributed as chi-square (K).

e. $Q^2(K)$ is the Ljung Box statistic identifying the presence of first order autocorrelation in the squared returns. Under the null hypothesis of no autocorrelation, it is distributed as chi-square (K).

f. ARCH LM statistic is the Lagrange Multiplier test statistic for the presence of ARCH .Under the null hypothesis of no heteroskedasticity, it is distributed as a chi-square (K). Critical value at 1 per cent level of significance is 6.63 at 1 degree of freedom. Values for other higher lag are also significant.

Descriptive statistics on S&P CNX Nifty returns is summarized in Table 1. For S&P CNX Nifty, the skewness statistic for daily returns is found to be different from zero indicating that the return distribution is not symmetric. Furthermore, the relatively large excess kurtosis suggests that the underlying data is leptokurtic or heavily tailed and sharply peaked about the mean when compared with the normal distribution. The Jarque-Bera statistic calculated to test the null hypothesis of normality rejects the normality assumption. The results confirm the well-known fact that daily stock returns are not normally distributed but are leptokurtic and skewed.

UNIT ROOT TESTS

Stationarity of the return series were tested by conducting Dickey-Fuller test. The calculated p-values of ADF were less than 0.005 which leads to a conclusion that the data of the time series under study is stationary. The results of both the tests confirm that the series are stationary. Table 2 presents the results of these tests.

Table 2

Unit Root Testing of Daily Returns

		t-Statistic	Prob.
Augmented Dickey-Fuller test statistic		-55.66558	0.0001
Test critical values :	1% level		-3.432036
	5% level		-2.862170
	10% level		-2.567149

Source : Calculated from the data taken from NSE website for selected period.

APPLICATION OF BOX-JENKINS METHODOLOGY

Volatility clustering implies a strong autocorrelation in squared returns; therefore, a simple method for detecting volatility clustering is to calculate the first-order autocorrelation coefficient in squared returns. To test this hypothesis, one can use the modified Box-Pierce (Ljung-Box- Pierce or simply Ljung-Box) statistic (Q), developed by Ljung and Box, which is defined as $Q = n\ (n + 2)\ \Sigma\ r^2\ k\ /\ (n - k)$, where n = sample size and k = lag length and $\Sigma\ r^2k$ = autocorrelation function (Ljung and Box, 1978). In an application, if the computed Q exceeds the critical Q value from the Chi-square table at the chosen level of significance, one can reject the null hypothesis. The values of $Q^2(1)$ test statistic (reported in Table 1), reject the null hypothesis which confirmed the presence of first order correlation in the series. The existence of a leptokurtic distribution, presence of volatility clustering suggested an

ARCH or GARCH process, which was confirmed by computing the value of Lagrange Multiplier (LM) which rejects the null hypothesis. To sum up , analysis indicates that the daily return series of the index is non-normal and exhibits 'ARCH effect'.

Fitting GARCH (1, 1) Model

We, therefore, use GARCH (p,q) model, the most popular member of the ARCH class of models, to model volatility of S&P CNX Nifty returns. EViews 5 software for model estimation has been used. The results from the model estimated are reported in Table 3.

Table 3

Coefficients of GARCH model

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001115	0.000235	4.754022	0.0000
RET(-1)	0.090352	0.018196	4.965642	0.0000
	Variance Equation			
C	8.37E-06	8.62E-07	9.715779	0.0000
RESID(-1)^2	0.133295	0.007104	18.76214	0.0000
GARCH(-1)	0.846834	0.006323	133.9330	0.0000

R-squared	0.001024	Mean dependent var	0.000474
Adjusted R-squared	-0.000122	S.D. dependent var	0.017526
S.E. of regression	0.017527	Akaike info criterion	-5.477936
Sum squared resid	1.070930	Schwarz criterion	-5.469116
Log likelihood	9566.737	F-statistic	0.893227
Durbin-Watson stat	2.051285	Prob (F-statistic)	0.467000

Source : Compiled with EViews 5

While running GARCH (1,1) process, we get the following estimated conditional variance equations :

$$h_t = \omega + \alpha_1\, \varepsilon_{t-1}{}^2 + \beta_1 h_{t-1}$$

$$h_t = 8.73E{-}06 + .133\, \varepsilon_{t-1}{}^2 + 0.846\, h_{t-1}$$

For time series analysis, it is desirable to have stationary series. Stationarity of the series can be found by summation of $\alpha_1 + \beta_1$ and the value of summation should be less than one. As for the stationarity of the variance process, it was observed that $\alpha_1 + \beta_1$ is 0.979 for S&P CNX Nifty (value of α_1 is +0.133 and the value of β_1 is +0.846 reported in Table 3). Hence, Stationarity condition ($\alpha_1 + \beta_1 < 1$) is satisfied here. However, the sum was rather close to one which indicated a long persistence of shocks in volatility implies a 'long memory'. A large value of GARCH lag coefficients β_1 (+0.846) indicates that shocks to conditional variance takes a long time to die out, so the volatility is 'persistent'. Low value of error coefficient α_1 i.e., (+0.133) suggests that market surprises induce relatively small revisions in future volatility.

Forecasts of Market volatility

Once the model has been fitted to market return series, it can be used to forecast volatility. We use the model to forecasts volatility for one day ahead.

$$h_t = \omega + \alpha_1\ \omega_{t-1}^{\ 2} + \beta_1 h_{t-1}$$
$$h_t = 8.73E - 06 + .133\ \varepsilon_{t-1}^{\ 2} + 0.846\ h_{t-1}$$
$$h_{t+1} = 8.73E - 06 + .133\ (-.00296)^2 + 0.846\ (9.07E - 05)$$
$$= .00008663$$

Where h_{t+1} is the one-day-ahead volatility forecast. The value of $\varepsilon_{t-1}^{\ 2}$ and h_{t-1} are calculated by using E Views 5. The value of one-day-ahead volatility is .00008663.Hence,it can be easily forecast one-day-ahead volatility from constructed model.

FINDINGS

The findings of the study suggest that the Indian stock market have significant ARCH effects and it is appropriate to use GARCH models to estimate the process. We, therefore, use GARCH (p, q) model, the most popular member of the ARCH class of models, to model volatility of Sensex returns. The model with large value of lag coefficient (+0.846) show that volatility in the NSE is highly persistent, i.e., long memory and is predictable. The relatively small value of error coefficient (+0.133) of GARCH (1,1) implies that market surprises induce relatively small revisions in future volatility.

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